



St Hilda's
ANGLICAN SCHOOL FOR GIRLS

Time:	30 minutes
Marks:	/29 marks
Total Marks:	/43 marks

12 Methods Topic Test 1

4 March 2021

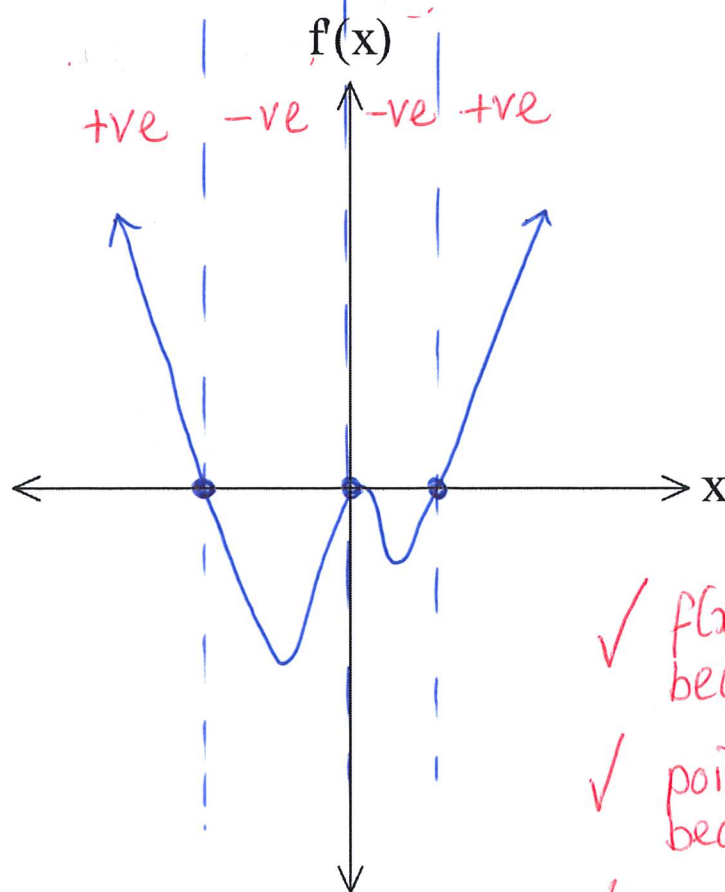
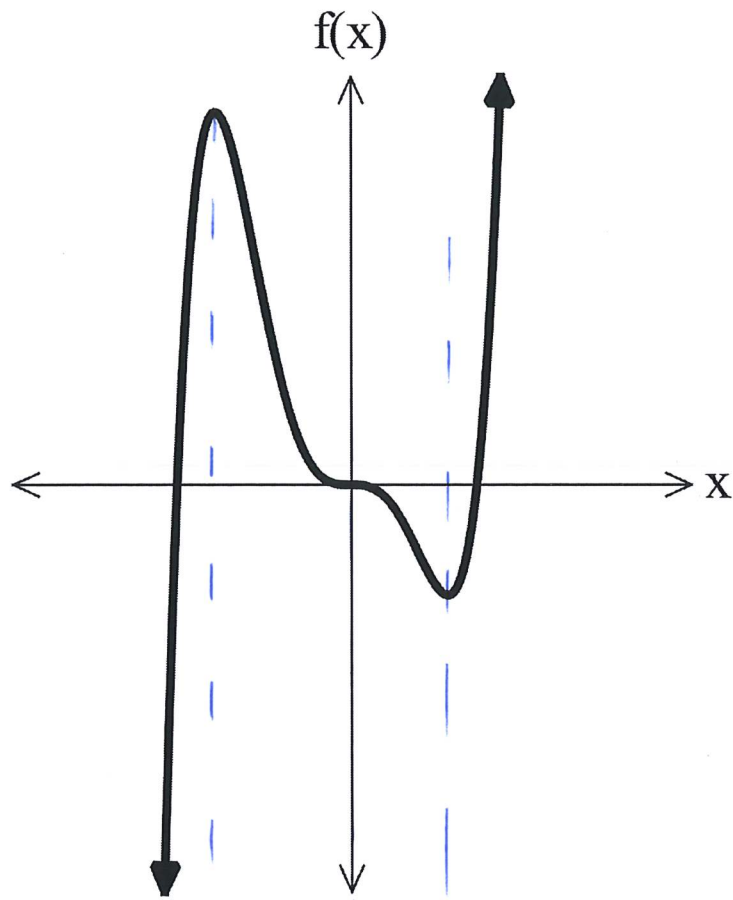
Resource Free

**ClassPad calculators are NOT permitted.
Formulae Sheet is permitted.**

Name: Solutions

1. [3, 2 = 5 marks]

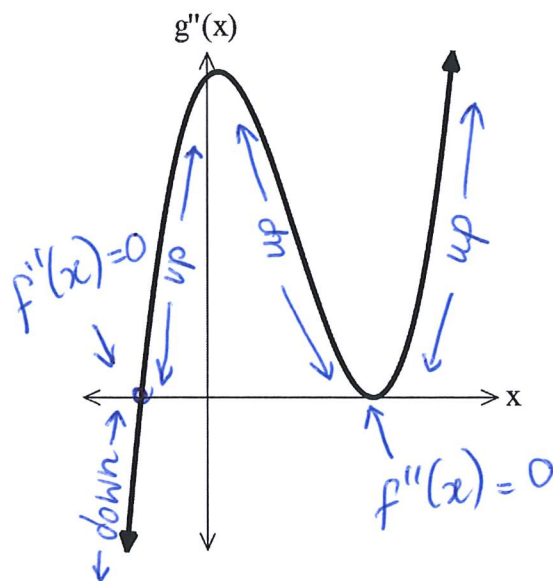
a) Sketch the graph of the first derivative of the following function:



- ✓ $f(x)$ stationary pts become roots of $f'(x)$
- ✓ points of inflection become local min/max
- ✓ general shape

b) Here is the graph of a function's second derivative.

concavity of $f(x)$
described in blue
ink on graph of $f''(x)$



How many points of inflection does the original function have?
Justify your answer.

✓ $f''(x) = 0$ mentioned

✓ one point of inflection

2. [2, 3, 3 = 8 marks]

a) If $y = x^{\frac{1}{3}} - \frac{2}{x}$ find $\frac{dy}{dx}$. (No need to simplify your answer.)

$$y = x^{\frac{1}{3}} - 2x^{-1}$$

$$\frac{dy}{dx} = \underbrace{\frac{1}{3}x^{-\frac{2}{3}}}_{\checkmark} + \underbrace{2x^{-2}}_{\checkmark}$$

b) $\frac{d}{dx}(4x\sqrt{x^3-9})$ (No need to simplify your answer.)

$$\begin{aligned} & \frac{d}{dx} [4x(x^3-9)^{\frac{1}{2}}] \\ &= \underbrace{4(x^3-9)^{\frac{1}{2}}}_{\checkmark} + \underbrace{4x \left[\frac{1}{2}(x^3-9)^{-\frac{1}{2}} \right]}_{\checkmark} \underbrace{(3x^2)}_{\checkmark} \end{aligned}$$

c) If $g(x) = \frac{2x}{(1-6x^2)^3}$ find $g'(0)$.

$$g'(x) = \frac{2(1-6x^2)^3 - 2x(3)(1-6x^2)^2(-12x)}{[(1-6x^2)^3]^2} \checkmark$$

$$g'(0) = \frac{2(1-6(0^2))^3 - 2(0)(3)(1-6(0^2))^2(-12(0))}{[(1-6(0^2))^3]^2}$$

$$= \frac{2}{1}$$

$$= 2 \checkmark$$

8

3. [7 marks]

For the function $y = x^3(4 - x)$:

a) Use calculus to determine the coordinates of any points of inflection on the curve and justify whether it/they are horizontal or not.

b) Describe the behaviour of the function as $x \rightarrow \infty$.

a) $y = 4x^3 - x^4$

$$\frac{dy}{dx} = 12x^2 - 4x^3 = 0 \checkmark$$

$$4x^2(3-x) = 0$$

$$x=0$$

$$x=3$$

$$y=0$$

$$y=27$$

$$\frac{d^2y}{dx^2} = 24x - 12x^2 = 0 \checkmark$$

$$12x(2-x) = 0$$

$$x=0$$

$$x=2$$

$$y=0$$

$$y=16$$

When $x=3$, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} \neq 0 \Rightarrow$ not pt of inflection

When $x=2$, $\frac{dy}{dx} \neq 0$, $\frac{d^2y}{dx^2} = 0 \Rightarrow$ non-horizontal pt of inflection

When $x=0$, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} = 0 \Rightarrow$ need sign test

x	-1	0	1
$\frac{dy}{dx}$	16	0	8
shape	/	—	/

horizontal pt of inflection

conclusion $\left\{ \begin{array}{l} (0,0) \text{ is a horizontal point of inflection.} \\ (2,16) \text{ is a non-horizontal point of inflection.} \end{array} \right.$

\checkmark evidence \checkmark evidence

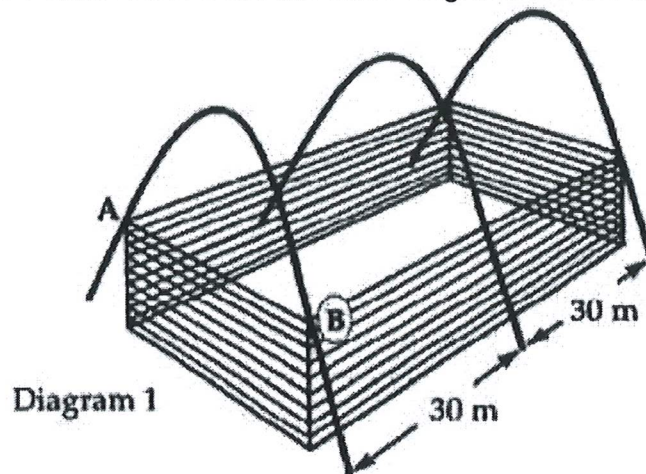
\checkmark y-coords

b) as $x \rightarrow \infty$, $y \rightarrow -\infty \checkmark$ (Working not req.)

when $x=100$, $y = 100^3(4-100) = -96\,000\,000$

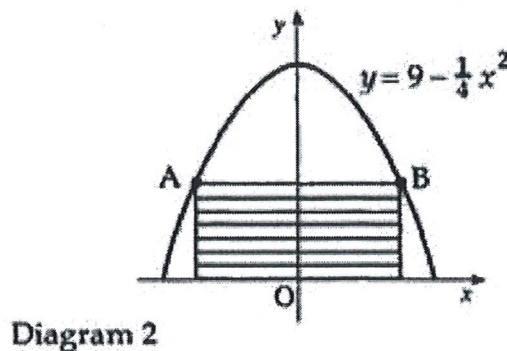
4. [1.8 = 9 marks]

An artist's impression of a new warehouse, based on the architect's plans, is shown in Diagram 1 below. Points A and B are the same height above the base.



The warehouse is in the shape of a cuboid and is supported by three identical parabolic girders spaced 30 metres apart.

With the coordinate axes as shown below in Diagram 2, the shape of each girder can be described by the equation $y = 9 - \frac{1}{4}x^2$.



- a) Given that AB is $2x$ metres long, show that the shaded area in Diagram 2 is $(18x - \frac{1}{2}x^3)$ square metres.

As $AB = 2x$ and both $y = 9 - \frac{1}{4}x^2$ and AB are symmetrical about the y -axis, x -coord of $A = -x$
 x -coord of $B = x$

$$\begin{aligned} \text{Area} &= AB \times y\text{-coord at } B \\ &= 2x \left(9 - \frac{1}{4}x^2\right) \checkmark \\ &= 18x - \frac{1}{2}x^3 \end{aligned}$$

(2)

- b) The architect wants to fit the rectangular prism (cuboidal) of maximum volume into the girders. Use calculus to find the value of this maximum volume, in simplest form.

$$\begin{aligned} V &= \text{shaded area} \times 60 \\ &= \left(18x - \frac{1}{2}x^3\right) \times 60 \\ &= 1080x - 30x^3 \quad \checkmark \end{aligned}$$

$$\frac{dV}{dx} = 1080 - 90x^2 = 0 \quad \checkmark$$
$$90(12 - x^2) = 0$$

$$90(\sqrt{12} - x)(\sqrt{12} + x) = 0 \quad \checkmark$$

$$x = \sqrt{12}$$

$$x = -\sqrt{12} \quad \text{discard} \quad \checkmark$$

$$\frac{d^2V}{dx^2} = -180x \quad \checkmark$$

When $x = \sqrt{12}$: $\frac{d^2V}{dx^2} = -180\sqrt{12} < 0 \Rightarrow$ Max TP
concave down \checkmark

When $x = \sqrt{12}$:

$$\begin{aligned} V &= 1080\sqrt{12} - 30(\sqrt{12})^3 \quad \checkmark \\ &= 1080 \times 2\sqrt{3} - 360 \times 2\sqrt{3} \\ &= 2160\sqrt{3} - 720\sqrt{3} \\ &= 1440\sqrt{3} \text{ m}^3 \quad \checkmark \end{aligned}$$

[END OF QUESTIONS]



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Time: 15 minutes
Marks: /14 marks

12 Methods Topic Test 1

12 March 2020

Resource Assumed

Formulae Sheet, half an A4 size page of notes and ClassPad calculators are permitted.

Name:

5. [3 marks]

If $y = 2x^3 - 7x$, use differentiation to find the approximate change in y when x changes from 5 to 4.99.

$$\frac{dy}{dx} = 6x^2 - 7 \quad \checkmark$$

$$\delta x = -0.01$$

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

$$= [6(5)^2 - 7](-0.01)$$

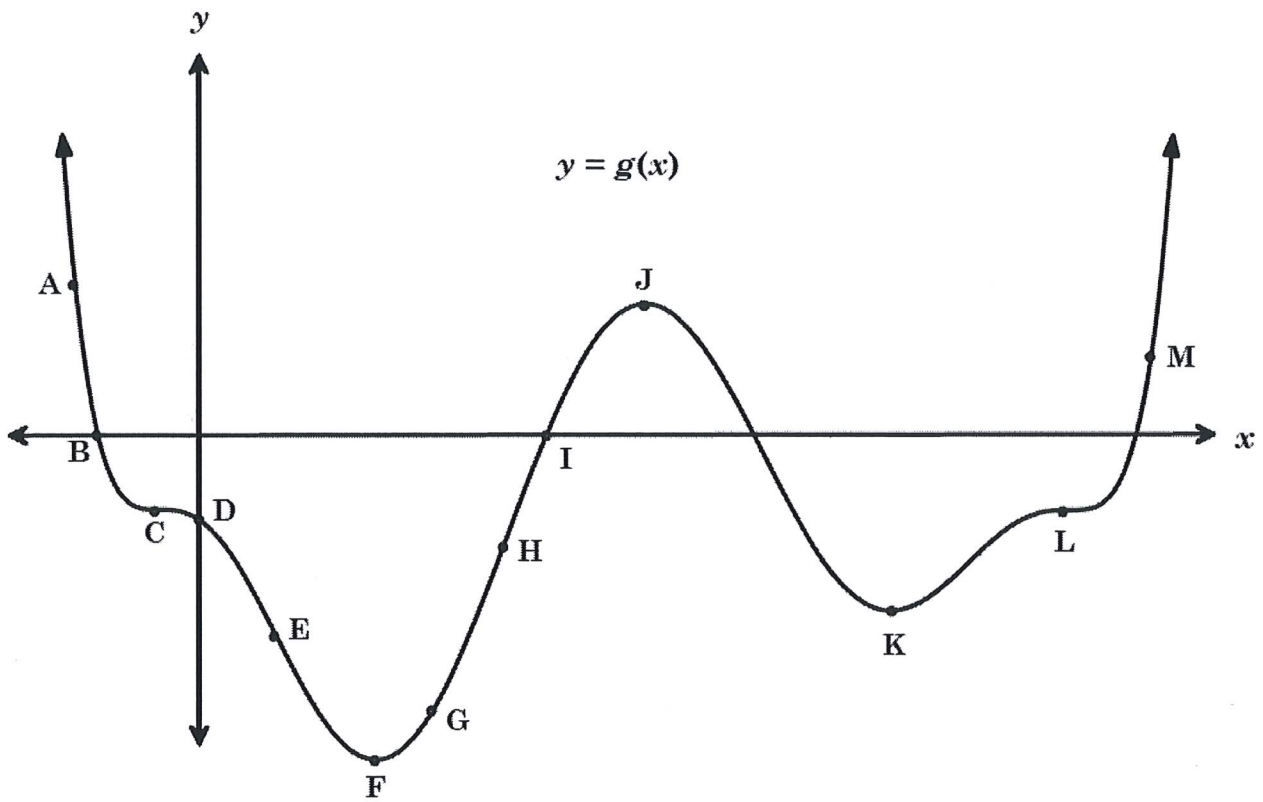
$$= -1.43 \quad \checkmark$$

$\therefore y$ will decrease by approximately 1.43
 \checkmark interprets -1.43

(3)

6. [6 marks]

The graph of $y = g(x)$ is shown on the axes below. Identify all points, from those labelled (A to M), which have each of the following properties. Write "none" if no such points exist.



- a) $g(x) = 0$ and $g'(x) > 0$ I ✓
- b) $g(x) = 0$ and $g''(x) > 0$ B ✓
- c) $g'(x) = 0$ and $g''(x) = 0$ C, L ✓✓
- d) $g'(x) = 0$ and $g''(x) < 0$ J ✓
- e) $g'(x) < 0$ and $g''(x) = 0$ E ✓

7. [2 marks]

Find the maximum and minimum values of the function $f(x) = x^3 + x^2 - 8x - 5$, for $-4 \leq x \leq 2$.

using ClassPad:

maximum is 7 ✓

(when $x = -2$)

minimum is -21 ✓

(when $x = -4$)

8. [2, 3 = 5 marks]

A particle starts at the origin and travels in a straight line with velocity v m/s given by $v = (5t - 2)(t + 1)$, $t \geq 0$.

a) Calculate the initial acceleration of the particle.

$$a = \frac{dv}{dt} = 10t + 3 \quad \text{when } t=0: a = 3$$

\therefore initial acceleration is 3 m/s^2 ✓

b) Is the particle speeding up or slowing down when $t = 0$? Justify your answer.

when $t=0$: $v = -2 \text{ m/s}$ ✓

as $v = -2$ and $a = 3$,

at $t=0$ the particle is moving to the left ✓ but acceleration is in the opposite direction, so particle is slowing down. ✓

[END OF QUESTIONS]

5